The problem of non-existence

in the framework of symbolic logic

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Introduction and apology

This study will mainly focus on classical logic, and the discussion is not complete, because it does not extend to quantified modal logic-based investigations of the topic. That framework has been omitted because quantified modal logic can be regarded as a questionable formal logic. In fact, the study will discuss only one existential sentence: (1) Cerberus is barking now. We know that (2) Cerberus is a three-headed dog that guards the gates of Hades, and we also know that (3) there are no three-headed animals at all. It is manifested that following from (2) and (3), “Cerberus does not exist at any time” (4). It is also common sense that the sentences “Cerberus is barking now” and “Cerberus is not barking now” equally entail that “Cerberus exists now” (5). The existence of Cerberus is a presupposition of any statement about Cerberus either barking or not barking now. The problem with which this study is concerned has a long literature in logic and linguistics. The antinomy is that the presupposition of any claim about Cerberus — such as that it is barking now — contradicts the fact that Cerberus does not exist at any time, and thus, it does not exist now either. However, what then is the logical value of (1)? Is it true, false, or neither? It would be true only if there is some kind of truth bearers that support (5), but I am sitting in my room and not in Hades, and I am very sure that there are no three-headed dogs, so (5) must be not true. Can we say, then, that it is false? Or should we rather say that it has no truth value because it does not express a proposition?

By applying formal logic language, it is reasonable to introduce a few simple notations. Let C be the bundle of essential properties of Cerberus; c:= Cerberus, $c=\iota x C(x)$, where "ι" is the usual descriptor symbol; R:= three headed; B(x,t)= x barks at t time; and B(c,now)= Cerberus is barking now. We know that $\forall x(C(x) \rightarrow R(x))$ and as long as Cerberus does exist, we suppose that $\forall x(C(x) \leftrightarrow x=c))$; $c = \iota x C(x)$, where "ι" is the usual descriptor symbol. Let us then look at the most promising suggestions for formulating the problem.

On Russell – Quine's way out

Following Russell's analysis, $B(c,now) \Leftrightarrow B(\iota x C(x),now) \Leftrightarrow \exists x(B(x,now) \& \forall y(C(y) \leftrightarrow y= x))$. In the spirit of Russell's description theory, the interpreted formula "$\exists x(B(x,now) \& \forall y(C(y) \leftrightarrow y=x))$" expresses the meaning — at least, the truth conditions of the statement — "Cerberus is barking now." In this way, it can be demonstrated that "Cerberus is barking now" is false.

(1) $\forall x(C(x) \rightarrow R(x))$

If something is Cerberus, then that animal is three headed.

(2) $\neg \exists x R(x)$

(3) $\neg \exists x C(x)$ (1)(2)

(4) $\neg \exists x \forall y(C(y) \leftrightarrow y = x))$ (3)

(5) $\neg \exists x(B(x,now) \& \forall y(C(y) \leftrightarrow y = x))$ (4)

Thus, by applying Russell's description theory, the sentence "Cerberus is barking now" is false. However, this analysis is not conclusive: there are other approaches to solve the problem.

The physicalist approach

Following Quine, we may think of Cerberus "over a period as a sum of the temporally small parts which are its successive momentary states." (Method of Logic, London, Routledge & Kegan Paul, 1958, p.210). Let us then define Cerberus as a special relation:

$C(x,t)$:= x is Cerberus at t moment

(1) $\forall x \forall y \forall t((C(x,t)\ \&\ C(y,t)) \rightarrow x{=}y)\ \&\ \forall x \forall t\ (C(x,t) \rightarrow R(x,t))$

If something is Cerberus at any time, then it is three headed at that time.

(2) $\neg\exists t \exists x R(x,t)$

(3) $\neg\exists t \exists x C(x,t)$ (1)(2)

Cerberus never exists.

(4) $\neg\exists t \exists x \forall y(C(y,t) \leftrightarrow y = x)$ (3)

(5) $\neg\exists t \exists x(B(x,t)\ \&\ \forall y(C(y,t) \leftrightarrow y{=}x))$ (4)

(6) $\neg\exists x(B(x,now)\ \&\ \forall y(C(y,now) \leftrightarrow y{=}x))$ (5)

There is no such unique thing that is Cerberus and that is barking now.

Formula (5) precludes only the existence of Cerberus in time, which means that there is no such function of time where the value series of the function is the life story of Cerberus.

Wyman's physicalist approach

Formula (1) says that if Cerberus has any property at t time, then it is a three-headed animal at that time.

(1) For any interpretation of F predicate, $\forall t(F(c,t)\rightarrow R(c,t))$

(2) $\forall t(B(c,t)\rightarrow R(c,t))$ (1)

(3) $\neg\exists t\exists xR(x,t)$

(4) $\forall t\forall x\neg R(x,t)$ (3)

(5) $\neg R(c,now)$ (4)

(6) $\forall t\neg B(c,t)$ (2)(5)

Cerberus is never barking.

(7) $\neg B(c,now)$ (6)

This yields a similar result as the previous two conclusions, but Wyman does not have to ensure the reference of the "Cerberus" name in time: he may say that Cerberus exists out of time and outside physical reality. In this manner, the claim of the non-existence of Cerberus means denying any physical property of Cerberus in time. Wyman calls into question that the existence of numbers is similar to the existence of animals.

Second-order logic considerations

In first-order logic, "existence" is a property of predicates and not names. Thus, applying second-order logic does not help us:

(1) x – exists $\leftrightarrow \exists\alpha(\alpha(x)$

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(2) x – exists $\leftrightarrow x=x$ (1)

(3) $x=x$ (axiom)

(4) $\forall x$. x – exists (2) (3)

This is not a very fruitful result – Edward N. Zalta's equivalent definition: x -exists:= $\exists$y(y=x) is also rather insignificant – but there is another way to proceed. We can generalize Wyman's approach to devise a tentative definition of existence in time:

(5) a exists at t time := $\Sigma\alpha(\alpha(a,t)$ & α is a measurable physical property), where "a" is an individual constant, "α" is a second-order variable, "Σ" is the symbol of an existential, and "Π" is the symbol of a universal substitutional quantifier.

One cannot substitute a predicate in (5), such as "x is identical with x at t time," due to the proviso of physical properties. There would be a similar proviso if one had to specify "α" in this way: α is a predicate of L physical language, where α is a range over the domain of predicate letters and not sets. This is the reason for the usage of substitutional quantification here, which is explained as follows.

Note on substitutional quantification

In the finite or denumerable infinite domain of discourse, the two kinds of quantification coincide. However, if our first-order domain is infinite, then its second-order domain is uncountable, and accordingly, there is no name for every second-order object. Furthermore, there are more sets than properties. The next two formulas explain the relation between objectual and substitutional quantifiers: $\Pi x\exists y.x=y$; $\neg\forall x\Sigma y.x=y$.

Metalanguage formulation

In the framework of classical logic, in every interpretation, an individual constant must refer to something in our domain of discourse, i.e., some fixed individual object. This means that empty names are banned in the first-order logic. This guarantees that for every "*F*" one-place predicate and "*a*" individual constant, $F(a) \Rightarrow \exists x F(x)$ and $\neg F(a) \Rightarrow \exists x \neg F(x)$ as well. Thus, if "*c*" is the symbol of Cerberus and "*c*" is an empty name because Cerberus does not exist, then "*c*" cannot be an individual constant of our formal logic language (R. Carnap used a special object — "null entity" as a reference to empty names — to eliminate the problem). It is easy to express in metalanguage that: "$F(c)$"$\notin WFF$ and similarly "$B(c,now)$"$\notin WFF$. At the metalanguage level, we can say that "*c*" is not an individual constant (name) of our physicalist language L_p, but "*c*" can be the name of a formalized Greek mythology language L_m. We can express it by applying different domains of discourse: $\neg\exists x(x=\delta(c)\,\&\,x\in physical\ reality)$ on the one hand and $\exists x(x=\delta(c)\,\&\,x\in Greek\ mythology)$ on the other hand ("δ" symbolizes the denotation function). It is not permitted to use mythological names — individual constants — in the framework of physicalist language, but it can be true or false that Cerberus is barking now in mythology. This is probably what Parmenides suspected: "VI. It needs must be that what can be thought and spoken of is; for it is possible for it to be, and it is not possible for what is nothing to be" John Burnet (1892, http://philoctetes.free.fr/parmenides.pdf).

At object language level, we cannot clearly distinguish between three sentences: (1) "Cerberus is not barking now"; (2) "It is not the case that Cerberus is not barking now"; and (3) "It is not true that Cerberus is not barking now." However, the distinction would be relevant to our topic because the three sentences are denoting something different. The first sentence is talking about an animal; the second is talking about a fact; and the third is talking about a sentence or proposition. The truth conditions of the three sentences are clearly different if we allow empty names. We may claim that (1) is not WFF or has no logical value, but in this case, (3) is still true. The metalanguage is an expedient framework to formulate this. However, what about (2)? Does (2) have the same meaning as (1) — as very often logical handbook claims — or does (2) resemble (3)?

Value gap framework

In strict terms, with the last step, we have crossed the border of classical logic and have ascended to the framework of value gap logic. We may claim that $2=|B(c,now)|$ or $2=|c=c|$, where "2" denotes the logical value gap. Unfortunately, this seemingly simple formal logic solution comes at a cost. In Kleene's value gap (viz. three valued) logic, classical logic equivalence does not hold, such as $p \Leftrightarrow p \,\&\, (q \vee \neg q)$ (not valid because of the evaluations of p and q: "$1 \Leftrightarrow 1 \;\&\; (2 \vee \neg 2)$"). In fact, the evaluation rule of universal and existential quantification as well as the concept of logical consequence have to be reconsidered in three-valued logic. This is an experimental definition for handling the empty name problem: let φ be a formula and Φ be a set of formulas; then, we may say that $\Phi \Rightarrow \varphi$ if and only if every interpretation and evaluation that generates is not a false logical value, as all Φ formulas also result in the true logical value of φ formula. In this manner, "$\neg F(a)$" does not entail "$\exists x. \neg F(x)$" in three-valued logic (Arthur Prior was aware of these problems).

Epilogue

Each approach has its pros and cons, including the usage of the null entity, but it is presumed that an acceptable formalization must work without intentional operators or the metaphysics of possible worlds. These are my thoughts on “what is not there.”